The Algebra of Jeremiah Day

Jeremiah Day was certainly an excellent expositor, as can be inferred from the nine-page "Introductory observations on the Mathematics in general." Here he lists 22 Articles (themes) that explain the nature of mathematics, which he defines as "the science of quantity." Article 2 states, "Those parts of Mathematics on which all others are founded are Arithmetic, Algebra, and Geometry," which the next three articles define, respectively, as "the science of numbers," "a method of computing principally by letters," and "the part of mathematics which treats of magnitude." Next, Day praises pure mathematics for its "clearness and distinctness of its principles … the exactness of definitions, axioms, and the demonstrations." He then added that mathematics is also valuable for applications, whereupon he provides a laundry list of fields with critical uses: mercantile transactions, surveying, mechanics, architecture, fortification, gunnery, optics, astronomy, history, government, chemistry, mineralogy, music, painting, and sculpture.

The contents of *Algebra* are what one would expect to find in high-school or college algebra courses today. For instance, Day shows how to solve $\sqrt{a^2+\sqrt{x}} = \frac{3+d}{\sqrt{a^2+\sqrt{x}}}$ and $a + dx^n = 10 - x^n$ for *x*. There are also the *de rigueur* word problems that seem to stump so many beginning students: "If you add 10 years to my age, and extract the square root of the sum, and from this root subtract 2, the remainder will be 6. What is my age?" But, like all books, one must read with care, because an example that exhorts the student to solve $\sqrt[3]{x+1} = 1$ supplies an answer of $x = 65$, which follows from the intermediate step $x + 1 = 4^3 = 64$. Can you locate the two typographical errors? [Spoiler alert: the two typos are $x = 63$ and $\sqrt[3]{x+1} = 4$.]

Two topics in Day's *Algebra* deserve attention. For one, he provides a chapter on Mathematical Infinity that could serve as an excellent introduction to calculus, discussing the notions of the infinitely large and the infinitesimally small in an intuitive manner, without mentioning limits. This chapter precedes coverage of division by compound divisors, where the concept of infinity enters at a critical juncture. A later chapter in the 296-page book introduces infinite series. Here Day takes pains to distinguish convergent from divergent sums, using an approach based on algebra and not limits, which would have been a revolutionary step forward at that time anyway, as it would have preceded the historic discoveries of Bernhard Bolzano (1781-1848) and Augustin-Louis Cauchy (1789-1857) by several years. To cite an example in Day, he found that $\sqrt{a^2 + b^2} = a + \frac{b^2}{2a} - \frac{b^4}{8a^3} + \frac{b^6}{16a^5}\ \&c$, and then applied it to conclude that

$\sqrt{2} = \sqrt{1+1} = 1 + \frac{1}{2} - \frac{1}{8} + \frac{1}{16}\ \&c.$