Analyst Contributors

This file presents some facts about the lives and accomplishments of seven members of the publication community from the journal, the *Analyst*. The most accomplished was Asaph Hall, though most of his 23 contributions were in observational and mathematical astronomy. The next two gained notorieties for work done on mathematical statistics, and were among the top five contributors to the journal (as measured by the number of entries in its index). Erastus De Forest had 26 entries and Charles Kummell 18. The remaining four figures were cited less frequently, and are of secondary rank but worthy of mention nonetheless: Edward Hyde (17 entries), Daniel Kirkwood (5), Mansfield Merriman (4), and Orson Pratt (4).

Asaph Hall III (1829-1907) was the most celebrated of the seven, as indicated by a long memorial article on his life and accomplishments by one of the leaders of 19th-century American mathematics, William Hill, shortly after Hall's death at age 87. Most of Hall's legacy lies in astronomy, and most of his publications were in this field, but he was also proficient at mathematics.

Asaph Hall was a Brahmin Connecticut Yankee, with roots stretching back to 1639, when an ancestor arrived in New Haven. His paternal grandfather, Asaph Hall I, was a Revolutionary War officer and state legislator. Hall III left school at age 16, three years after his father died. Hall was then apprenticed to a carpenter for three years, after which he became a journeyman carpenter. Because such work was rare in winter, he set about studying algebra and Euclidean geometry. All the while, by the fall of 1854 he had saved enough money to attend Central College in McGraw (NY) because of its low tuition and students could work to pay their way through manual labor. Although he wanted to study mathematics, no mentor at the time was able to teach him. Therefore, he left after only a year and a half. But during his first semester, he met Angeline Stickney, who graduated in 1855, joined the faculty, and became his geometry instructor. They married a year later. Central College was famous for educating both blacks and whites, including Charles L. Reason, but it closed in 1860 due to financial difficulties and a smallpox epidemic.

After marrying, the Halls moved to Ann Arbor, where he entered the University of Michigan in the sophomore class and excellent in astronomy. However, the couple left after only three months and moved to Ohio, where they ran a school 1856-1857. During that time, Hall studied astronomy and mathematics whenever he could. The next year, she moved back home to teach school while he moved east with the intention of enrolling at Harvard. After working some carpentry jobs in the area to save money, he was finally able to attend Benjamin Peirce's lectures, all the while finding employment at the Harvard Observatory under W.C. Bond. His wife then rejoined him. But he did not attend Peirce's lectures very long because he found them too theoretical. Besides, there was tension between the mathematics department and the Observatory.

Over the next six years, Hall became an expert a computing orbits of various celestial bodies, leading to his appointment as assistant astronomer at the Naval Observatory in 1862 in Washington DC, even though the U.S. was in the midst of the Civil War. He was promoted to professor the following year, and remained at the Observatory until retiring in 1891 at age 62. In 1877, he discovered the two moons of Mars, Phobos and Deimos, for which he is mainly known today. Hall's wife died one year after his retirement, and four years after that, 1896, he accepted an invitation from Harvard to lecture on celestial mechanics. He delivered such lectures for five years, when he retired for the second, and final, time and moved back to his hometown of Groten (CT). By early 1907 he found it necessary to move into the home of his son, Angelo, a professor of mathematics at the Naval Academy. Hall died that November in Annapolis.

The Hill memoir on Hall contains a complete list of Hall's publications. The range of outlets for his works ran over several mathematics journals in the second half of the nineteenth century. His earliest paper on a mathematical topic appeared in Runkle's *Mathematical Monthly* in 1861, and dealt with the transformation of an infinite series into a continued fraction.[1] Earlier in the journal, he had won two prizes for solutions to posed problems.

During 1867-1871, he published four mathematics papers in Artemas Martin's *Messenger of Mathematics*, including one on a method for approximating the value of π.[2] This paper was notable for suggesting the Monte Carlo method. In it, Hall described an experiment in random sampling he had persuaded a friend to perform while recuperating from wounds. The experiment involved repetitively throwing at random a fine steel wire onto a plane wooden surface ruled with equidistant parallel lines, and resulted in

$$\pi \approx \frac{2ml}{an}$$

where $m =$ number of trials, $l =$ length of the steel wire, $a =$ distance between parallel lines, and $n =$ number of intersections. This experiment precisely matched the experiment that Le

Clerc de Buffon conducted in 1733 and is called Buffon's needle problem today. Both were early uses of the type of random sampling known as the Monte Carlo method since World War II.

Even beyond his contributions to the *Mathematical Monthly*, the *Analyst* contained numerous problems posed, and solved, by Asaph Hall. Two indicate how far American mathematicians had advanced since problems posed in the *Mathematical Correspondent* 70 years earlier. One, for instance, called for evaluating the determinant of a general 4×4 matrix. Another required a proof that

$$\int_0^a dx \int_0^x \varphi(x,y)\, dy = \int_0^a dy \int_y^a \varphi(x,y) dx$$

His first paper, on comets and meters, was the initial article in the second issue of the journal in 1874. That volume also contained a mathematics paper on the Besselian function.[3] Two years later, Asaph Hall presented clever methods of numerical integration that are appropriate for enrichment projects in a Calculus II class.[4]

The Hall paper I found most useful for telling students about the kind of discipline necessary for making progress in mathematics appeared in 1881. He recalled that, in 1858, two years after reaching the Harvard Observatory, he began reading a translation of the *Theoria Motus* by Karl Friedrich Gauss:[5]

> I well remember that at first the style of the work fairly took me off my feet, and seemed to leave me dangling in the air for a month or two before … the beauty and power of Gauss' methods were seen and felt. Having no teacher nor any one to assist me, I made it a rule to work out every equation and all the numerical examples before going on. … The whole reading occupied me nearly a year.

Hall's discipline paid dividends that year, resulting in two corrections to *Theoria Motus* that he published in the *Mathematical Monthly*. The aim of the *Analyst* paper, Hall wrote, was to provide ten "of the points and reductions that gave me the most trouble." He concluded the paper with advice for prospective readers of the classical memoir: "It was by keeping the problem steadily before his mind for several years, and carefully working out all its parts, that Gauss brought his solution at last to a form almost perfect."[6]

The first of the two mathematical statisticians in the *Analyst* publication community was Erastus De Forest, who seems to have benefitted the most from *The Analyst*. He is an interesting character too from a personal standpoint, as his life's story might resonate with undergraduate students today.

Like Asaph Hall, **Erastus Lyman De Forest** (1834-1888) was an aristocratic Connecticut Yankee with family roots in the New World stretching back to the seventeenth century (1623). By the nineteenth century the family had acquired considerable wealth and his father was a physician who had graduated from Yale. Erastus De Forest enrolled at Yale at age 16, and when he received a B.A. four years later, his father endowed the De Forest Mathematical Prize at the university. Erastus De Forest then studied engineering at Yale's Sheffield Scientific School for the next two years, earning his Ph.B. in 1856. During this time Willard Gibbs was a student in engineering and Hubert Newton a young faculty member at Yale.

Like many students today, De Forest was quite unsure of his direction in life at graduation. But graduates from wealthy families do not have to join the work force at once; they have sufficient resources for taking time to travel to "find themselves." So, the next February, De Forest headed to Havana with an aunt. Shortly before sailing from New York, however, he disappeared, leaving his luggage and no clue of his whereabouts. Naturally the family panicked, their frantic state assuaged by neither speculation in the *New York Times* that their only child had met with foul play, nor by rumors that he had drowned in the East River. Yet the body was not found. Two years later, his father received a letter from him postmarked Australia. Erastus De Forest explained that he had been depressed and therefore headed to California, where he worked in the mines and taught public school for a year before continuing to Melbourne. From 1858 to 1860 he had been an assistant master at a Church of England grammar school, where he taught mathematics, surveying, and drawing.

De Forest's letter informed the family that he would be returning to Connecticut by way of India and England, and indeed the relieved parents greeted him warmly upon his return in 1861. Even though De Forest remained unsettled for the next two years while the Civil War raged about him, it was a two-year trip to Europe 1863-1865 that seems to have settled him down and set him on a career path in mathematical statistics and caring for his father; he never married. Over his subsequent career, Erastus De Forest became a notable statistician within the American mathematics community. He determined policy liabilities and worked on the problem of smoothing mortality tables for insurance companies. In two Smithsonian reports from the early 1870s, he introduced optimality criteria, interpolation, and smoothing problems. In the next decade, De Forest became a frequent contributor to the *Analyst*, as exemplified by Table 1, which lists the titles of his papers over the last five years of the Journal.

Year	Vol.	Title
1878	5	On the grouping of signs of residuals
1878	5	On repeated adjustments, and on signs of residuals
1878	5	On the limit of repeated adjustments
1879	6	On the development of $[p + (1 - p)]^x$
1879-80	6, 7	On unsymmetrical adjustments, and their limits (three parts)
1880	7	On some properties of polynomials
1880	7	On a theorem in probability
1881	8	Law of facility of errors in two dimensions
1881	8	On the elementary theory of errors
1882	9	Law of error in the position of a point in space
1882-83	9, 10	On an unsymmetrical probability curve (three parts)
1883	10	A method of demonstrating certain properties of polynomials

Table 1. Papers on statistics in the Analyst by E.L. De Forest

Even though the *Annals* succeeded the *Analyst* when it ended after Volume 10, De Forest chose to publish two important statistics papers in *Transactions* of the Connecticut Academy of Arts and Sciences in the mid-1880s.[7]

The other mathematical statistician in the *Analyst* publication community was Charles Hugo Kummell, with 18 entries in its index. The first was his initial foray into the mathematics publishing sphere in 1876 at age 40. Printed in two parts, the article showed that observational errors in an experiment are normally distributed.[8] He published an improved version of the proof of one result three years later.[9] A recent paper (2013) analyzed this important article and generally placed, in historical perspective, Kummell's contributions to the law of errors and to the least-squares method.[10] That paper singled out two other important aspects of this Kummell work: 1) the use of the quantity

$$h = \frac{1}{\epsilon\sqrt{2}}$$

as the measure of precision of a system of observations, and 2) the precisions of different systems of observations can be compared by means of probable error $r = 0.6745\epsilon$. (In modern terms, for normally distributed errors, the median absolute deviation is 0.6745 times the standard deviation).

Over his lifetime, Kummell published thirty papers, including several others in the *Analyst*. A three-part article dealt with Cauchy's theory of residues.[11] Two footnotes in the article indicate the problems that typesetters like Joel Hendricks had in typesetting. At the end of the second part of the article (p. 46), he added, "Mr. Kummell has contributed another § to this paper but for want of suitable type we are not able to insert it at present, but hope to be able to do so before the close of the present volume." When that third part did appear, he added (p. 175, fn. *), "For want of sorts, *k* is here … written instead of the Agate Greek π.—Compositor." Other topics that Charles Hugo Kummell dealt with in the pages of the Analyst were least squares,[12] differential geometry,[13] elliptic functions,[14] and geometry.[15]

Gottfried Wilhelm Hugo Karl Kummell (1836-1897) graduated from a polytechnic school in his native Germany at age 16 and then entered the University of Marburg, but left without taking a degree in early 1854. After teaching in Prussian schools, he left for Norfolk (VA) in 1866, whereupon he changed his name to **Charles Hugo Kummell**. He taught in Norfolk schools for the next five years before being appointed assistant engineer with the U.S. Lake Survey in Detroit in 1871. This agency had been established by Congress 30 years earlier to conduct surveys of the northern and northwestern lakes, as well as to prepare nautical charts and other aides for navigation. Kummell left the Lake Survey in 1880 and moved to Washington, DC, as a statistician and (human) computer with the U.S. Coast and Geodetic Survey. He remained in these positions until his death 17 years later.

After moving to the nation's capital, Kummell became active in an organization that was short-lived yet anticipated a national organization of mathematicians. The Philosophical Society of Washington (**PSW**) was founded in 1871 by Joseph Henry, the head of the Smithsonian Institution. Simon Newcomb served as one of its early presidents and J.J. Sylvester lectured on the theory of quaternions during his first semester at Johns Hopkins five years later. In 1883, the PSW formed a Mathematical Section with 35 members that included C.S. Peirce, George Hill, and Simon Newcomb, as well as Kummell. The aim of the Mathematics Section was to discuss papers in pure and applied mathematics. At its first meeting the chair, Asaph Hall, advocated founding a new mathematical journal, but his proposal came to naught. A short while later, Artemas Martin proposed forming a national organization of mathematicians. It was moved that a committee be formed to report on the advisability of establishing such an organization, but the measure lacked a second, so the matter was postponed indefinitely. The American Mathematical

Society (**AMS**) was founded in 1888, just four years after Martin's proposal. The Mathematics Section of PSW was disbanded in November 1892, probably because it no longer served any need due to the early success of the AMS.

It is somewhat surprising that Edward Hyde ranks so high on the *Analyst* entries list because he is perhaps the least known of all the mathematical practitioners named here. On the other hand, I feel that his accomplishments deserve more prominent attention than they have garnered.

The lifetime of **Edward Wyllys Hyde** (1843-1930) extended from the Peirce generation up to the onset of the founding of the Institute for Advanced Study at Princeton and the influx of European mathematicians fleeing Nazi atrocities. A native of Saginaw (MI), Hyde received a C.E. degree at Cornell in 1872, shortly after the university was founded. He served as an instructor on the faculty while a senior, and over the next year. He then joined the faculty at Pennsylvania Military College,[16] but left Chester in 1875 to become the first (assistant) professor of mathematics (simultaneously instructor in civil engineering) at the University of Cincinnati. The president of Cincinnati at the time was the mathematician Henry Turner Eddy. Hyde later served as dean of the College of Liberal Arts and president of the University (for three different terms in the 1890s), but was forced out in 1900 after a dispute with the new president Howard Ayers.

Edward Hyde was quite active in the American mathematical community that emerged around 1900. He served as an associate editor of the *Annals of Mathematics* 1896-1899. In addition, he was one of 28 signers of the 1896 circular "A call to a conference in Chicago" that led to the formation of the Chicago Section of the American Mathematical Society. The next year he was elected to the AMS Council for the term 1897-1899, along with Woolsey Johnson and B.O. Peirce.

Hyde wrote three books, the first one on civil engineering. *Skew Arches* was published in 1875 and became available on the Internet 125 years later. His other two influential books were on mathematics. *The Directional Calculus: Based upon the Methods of H. Grassmann* (Ginn and Co., 1890) was the first textbook on Grassmann's calculus in English. Called one "of Grassmann's [two] most important followers,"[17] Hyde published *Grassmann's Space Analysis* (Wiley, 1906) sixteen years later. These two books on vectorial analysis "contain Grassmann's ideas in a simplified form, i.e., limited to three dimensions and with the stress on applications."

The 1906 book was first published as a chapter in the *Higher Mathematics* by Mansfield Merriman and R.S. Woodward, which explains why its title page asserts "fourth edition."

The roots for Hyde's 1890 text appeared a decade earlier with a four-part paper "Mechanics by quaternions" that appeared in the *Analyst* in 1880 and 1881.[18]
He introduced the long and detailed series as follows (p. 137):

> Owing to the fact that certain of the quantities treated in Mechanics possess *direction* as well as *magnitude*, and are thus in their very nature *vector* quantities, it appears that the Quaternion methods should be peculiarly fitted for dealing with mechanical problems. Such is indeed the case, and it is proposed in these papers to give an elementary quaternion treatment of the subject. [Emphasis due to Hyde.]

This active journal author also published five papers on analytic geometry, two on synthetic geometry,[19] and one on integration. Curiously, his first article on analytic geometry lacked figures, but all four after that did.[20] The second paper proved a proposition that would make an appropriate project for an undergraduate course:

> Given four points [no one point lying within the triangle formed by the other three] to construct geometrically the axis and focus of the parabola passing through them.

In the paper on integration techniques,[21] Hyde presented a method for evaluating the triple integral

$$V = \int_{x=0}^{x=a} \int_{y=0}^{y=f(x)} \int_{z=0}^{z=\theta(x,y)} dx\, dy\, dz$$

by reducing it to a double integral.

The above-mentioned **Mansfield A. Merriman** (1848-1925) was a civil engineer born in Southington (CT) to parents with roots in early 17th-century America. Merriman graduated from the Sheffield Scientific School at Yale in 1871, and remained another year to earn a master's degree in civil engineering. He served as an assistant in the U.S. Corps of Engineers 1872-1873 and then spent six months studying in Berlin, Dresden, and Hanover. Upon returning to New Haven, he became an instructor in civil engineering at Sheffield 1874-1878. During that time, he earned a Ph.D. from Yale, in 1876, for the dissertation "Elements of least squares." In 1878, he was appointed professor of civil engineering at Lehigh University, where he remained until 1907, when he engaged in a consulting practice in New York. While at Lehigh, Merriman was also an assistant at the U.S. Coast and Geodetic Survey 1881-1885. All the while, he conducted research on strength of materials and the design of bridges, consulted on several civil- and hydraulic-engineering projects, and pursued mathematics. Merriman published several books in these engineering fields; within mathematics, his *Method of Least Squares* was published in 1884 and ran to eight editions.

In 1877, Mansfield Merriman published a long bibliography of 408 articles on the theory of errors and the method of least squares.[22] One of those articles was Charles Kummell's *Analyst* paper from the previous year [Endnote 7], and Merriman's comments caused the kind of bad blood between them that frequently happens when a reviewer criticizes an author's work. Initially, Merriman's comment seemed harmless: In the Kummell article, he wrote, "Hagen's proof of 1837 is given abbreviated and improved, and the usual rules for normal equations and probable error are deduced."[23] However, later in 1877 another paper by Merriman[24] asserted that the Kummell article:[25]

> although very abbreviated, and requiring in its readers a previous knowledge of the subject, is very welcome to mathematicians, and it contains one or two modifications of the German method of presentation, which considerably shortens the algebraic work.

Quick to take offense, Kummell replied:

> My paper is very abbreviated, as stated by Mr. Merriman, but is, nevertheless, clear and logical to any careful reader, and gives not a mere glimpse of the theory, but almost everything essential. Mr. Merriman's article contains a number of logical and theoretical blunders, which should not go uncorrected.

The matter got particularly testy when Kummell added, "Mr. Merriman writes that I have given Hagen's proof. Now who would like to be accused of such a thing?" Kummell claimed that his proof was totally original, and the controversy seems to have ended there.

Mansfield Merriman was essentially an engineer, as his first paper in the Analyst indicates. His introductory statement exhibited a typical attitude that separates engineers from mathematicians today: "As a matter of purely mathematical interest I wish to give here, without demonstration, the relations between the reactions of continuous girders of equal spans resting on level supports."[26]

The other three papers that Merriman published in the *Analyst* dealt with the method of least squares (**MLS**). The first dealt with the history of the MLS. Merriman wrote "The honor of the first publication of the method belongs to Legendre," from 1805.[27] He then cited 13 different proofs of the method, from the first by Adrain in 1808 to one by Crofton in 1870, including others by world renowned mathematicians Gauss (1809 and 1823), Laplace (1810), and Bessel (1838). It is curious that there was no mention of Kummell's paper published in the *Analyst* one year before Merriman's paper appeared in 1877.

Merriman did not have a copy of Adrain's 1808 paper from the earlier journal called the *Analyst* when he conducted his study of the MLS. However, later that year he wrote:[28]

> At the time the first paper appeared, in March, "I had not seen Adrain's original paper … having lately been able to consult a copy … I found that on pages 96 and 97, there is given a second deduction of the law of facility or error of an entirely different nature from that presented on pages 93-95. As this is a matter of considerable historical interest and as The Analyst for 1808 is quite rare I give the proof in Adrain's own words.

The author appended a page with data on the papers on the MLS he had catalogued in his initial study. Regarding American mathematicians' limited access to journals abroad, Merriman wrote:[29]

> With better library facilities the number of titles in the Italian, Dutch, and Scandinavian languages would be much increased; and one who can consult the Russian and Hungarian literature might undoubtedly find a few titles to add.

The first paper to appear in *The Analyst* was penned by **Daniel Kirkwood** (1814-1895), who was born in Maryland but educated at the York County Academy in Pennsylvania, was on astronomy.[30] He was a principal at two nearby academies 1843-1851, when appointed professor of mathematics at the University of Delaware. One year earlier he had earned a master's degree at Washington College (PA), and a year later, 1852, an LL.D. from the University of Pennsylvania. Daniel Kirkwood served as Delaware president 1854-1856 before his appointment as professor of mathematics at Indiana University. He remained in Bloomington for ten years before accepting the same post at Washington and Jefferson College (PA) for one year, whereupon he was recalled to Indiana and remained there until retiring in 1886. He then moved to Stanford as a lecturer, and lived in Palo Alto on an orange ranch for the rest of his life. An article in the first volume of the *American Mathematical Monthly* provides more details, as well as a list of his publications.[31]

Daniel Kirkwood published three other articles in the Analyst, one of which was also on astronomy.[32] His biography of the mathematician William Lenhart was cited in Chapter 3.[33] The remaining article was a short note on determining the length of a day.[34]

The remaining notable figure essentially published only one paper in the *Analyst*, but it led to three additional entries caused by yet more friction, this time between the editor and the author. **Orson Pratt** (1811-1881), then of Salt Lake City but born in central New York, is mostly known today for his role with the Church of the Latter Day Saints, from the time he was ordained by Joseph Smith in New York at age 20. While serving on numerous missions, he conducted an independent study of mathematics 1836-1844, leading to his appointment as instructor at the University of Nauvoo (IL) when it was formed in 1841. Reputedly, he taught calculus there. Six years later, Pratt was the scientific observer for the Vanguard Company, led by Brigham Young, that entered Salt Lake Valley as part of the cross-country campaign for Mormon colonization. Along the way he invented the odometer, which he called a "roadometer." It seems that Pratt wrote a calculus book in the 1850s, but no copies are extant.

Orson Pratt was only involved with the *Analyst* during 1876-1877. Initially he proposed a set of six problems regarding velocities and forces of orbiting bodies that was published in November 1876.[35] He supplied solutions in the next issue, January 1877, but at the end of those solutions, journal editor Joel Hendricks criticized Pratt's propositions as a basis for a theory of gravity for lacking a conceivable cause.[36] Pratt rebutted that assertion in the next issue, stating that his propositions had no bearing on, or reference to, the cause of gravity. Nonetheless, Hendricks dissented entirely once again,[37] thus putting an end to the 66-year-old Pratt's involvement with the journal.

Earlier, Orson Pratt must have sent Joel Hendricks a copy of his 1866 book *New and Easy Method of Solution of the Cubic and Biquadratic Equations*. Hendricks listed it in the *Analyst* but wrote only, "We are not at present prepared to speak of the merits of this book, but insert a single paragraph from the author's preface which will indicate its character."[38]

Endnotes:

[1] Asaph Hall, To change a series into a continued fraction, *Math. Month*ly **3** (1860), 262-268.

[2] Asaph Hall, On an experimental determination of π, *Messenger Math*. **2** (1873), 113-114.

[3] Asaph Hall, The Besselian function. *Analyst* **1** (1874), 81-84.

[4] Asaph Hall, Approximate quadrature. *Analyst* **3** (1876), 1-10.

[5] Both quotations in this paragraph appear on p. 83 of Asaph Hall, Notes on Gauss' *Theoria motus. Analyst* **8** (1881), 83-88.

[6] *Ibid*, p. 88.

[7] E.L. De Forest, On an unsymmetrical law of error in the position of a point in space, *Trans. Connecticut Acad. Arts Sci.* **6** (1884), 122- 138; and On the law of error in target-shooting, **7** (1885), 1- 8.

[8] Chas. H. Kummell, New investigation of the law of errors of observations, *Analyst* **3** (1876), 132-140, 165-171.

[9] Chas. H. Kummell, Revision of proof of the formula for the error of observation, *Analyst* **6** (1879), 80-81.

[10] See pp. 326-330 of Asta Shomberg and James Tattersall, Life and statistical legacy of Charles Hugo Kummell, *Mathematics Magazine* **86** (2013), 323-339.

[11] Chas. H. Kummell, An account of Cauchy's "Calcul des residues," *Analyst* **6** (1879), 1-9, 41-46, and 173-176.

[12] Chas. H. Kummell, Reduction of observation equations which contain more than one observed quantity, *Analyst* **6** (1879), 97-105; Proof of some remarkable relations in the method of least squares, *Analyst* **7** (1880), 84-88.

[13] Chas. H. Kummell, Some relations deduced from Euler's theorem on the curvature of surfaces, *Analyst* **8** (1881), 93-95.

[14] Chas. H. Kummell, "Remarks on Mr. Meech's article on elliptic functions" and "Evaluation of elliptic functions of the second and third species, *Analyst* **5** (1878), 17-19 and 97-104.

[15] Chas. H. Kummell, Approximate multisection of an angle and hints for reducing the unavoidable error to the smallest amount, *Analyst* **5** (1878), 172-174.

[16] I attended Pennsylvania Military College (PMC) for three years but left without a degree, so I never became an officer (or a gentleman). PMC became Weidner University in 1972.

[17] The two quotations in this paragraph appear on pp. 54 and 105 of Michael J. Crowe, *A History of Vector Analysis*, South Bend (IN): University of Notre Dame Press, 1967.

[18] E.W. Hyde, Mechanics by quaternions: *Analyst* **7** (1880), 137-144 and 177-184; *Analyst* **8** (1881), 17-24 and 49-55.

[19] E.W. Hyde, Proposition in transversals, *Analyst* **5** (1878), 113-115; Proof of a proposition in solid geometry, *Analyst* **7** (1880), 157-158.

[20] E.W. Hyde, Foliate curves, *Analyst* **2** (1875), 12-14; Solution of Mr. Church's problem, *Analyst* **2** (1875), 76-77; Proof that no two different ellipses can be parallel, *Analyst* **2** (1875), 144; The section of a circular torus by a plane passing through the center and tangent at opposite sides, *Analyst* **3** (1876), 78-79; To fit together two or more quadrics so that their intersections shall be a plane, *Analyst* **3** (1876), 97-99.

[21] E.W. Hyde, Limits of the prismodal formula, *Analyst* **3** (1876), 113-116.

[22] M.A. Merriman, A list of writings related to the method of least squares, with historical and critical notes, *Trans. Connecticut Acad. Arts and Sciences* **4** (1877) 151–232.

[23] As quoted on p. 330 of Shomberg and Tattersall, Life and statistical legacy. [Endnote 9.]

[24] M.A. Merriman, An elementary discussion of the principle of least squares, *Journal Franklin Inst.* **104** (1877) 270–274.

[25] The remaining quotations in this paragraph appeared on p. 330 of Shomberg and Tattersall, Life and statistical legacy. [Endnote 9.]

[26] On p. 138 of Mansfield Merriman, Note on the reactions of continuous beams, *Analyst* **2** (1875), 138-140.

[27] On p. 33 of Mansfield Merriman, On the history of the method of least squares, *Analyst* **4** (1877), 33-36.

[28] On p. 140 of Mansfield Merriman, On the history of the method of least squares, *Analyst* **4** (1877), 140-143.

[29] *Ibid*, p. 143.

[30] Daniel Kirkwood, On the relative positions of the asteroidal orbits, *Analyst* 1 (1874), 2-3.

[31] Robert J. Aley, Biography: Daniel Kirkwood**,** *Amer. Math. Monthly* **1** (1894), 140-149.

[32] Daniel Kirkwood, On the limit of planetary stability, *Analyst* **8** (1881), 1-3.

[33] Daniel Kirkwood, Reminiscences of William Lenhart, Esq., *Analyst* **2** (1875), 181-182.

[34] Daniel Kirkwood, On the variation of the length of the day, *Analyst* **7** (1880), 9-10.

[35] Orson Pratt, Sen., Six original problems, *Analyst* **3** (1876), 186-187.

[36] On p. 19 of Orson Pratt, Sen., Solutions to the six original problems published in No. six, Vol. III, *Analyst* **4** (1877), 15-19.

[37] *Analyst* **4** (1877), 55-56.

[38] Book notices, *Analyst* **3** (1876), 12.